

Physics 1A Discussion (Week 3)

Problem 1 (problem 2.62 on page 62 of the textbook)

The engineer of a passenger train traveling at 25.0 m/s sights a freight train whose caboose is 200 m ahead on the same track. The freight train is traveling at 15.0 m/s in the same direction as the passenger train. The engineer of the passenger train immediately applies the brakes, causing a constant acceleration of 0.100 m/s² in a direction opposite to the train's velocity, while the freight train continues with constant speed. Take $x = 0$ at the location of the front of the passenger train when the engineer applies the brakes.

(a) Will the cows nearby witness a collision?

The position of the passenger train changes in time as,

$$\begin{aligned}x_p(t) &= v_p(t=0)t + \frac{1}{2}a_pt^2 \\&= (25.0 \text{ m/s})t + \frac{1}{2}(-0.1 \text{ m/s}^2)t^2,\end{aligned}\tag{1}$$

Here, the origin of the x-axis position is set to be the initial position of the passenger train *i.e.* $x_p(t=0) = 0$ m. The position of the freight train changes in time as

$$x_f(t) = (15.0 \text{ m/s})t + 200 \text{ m}\tag{2}$$

When they collide at time T , we can say that

$$x_p(T) = x_f(T).\tag{3}$$

Therefore, we now set these equations equal to one another to see whether or not these paths intersect in the same space and time.

$$(25.0 \text{ m/s})T + \frac{1}{2}(-0.1 \text{ m/s}^2)T^2 = (15.0 \text{ m/s})T + 200 \text{ m}\tag{4}$$

$$-(0.05 \text{ m/s}^2)T^2 + (10 \text{ m/s})T - (200 \text{ m}) = 0 \text{ m}\tag{5}$$

Solving for the roots of this equation we get,

$$T_{\pm} = 100 \pm 20\sqrt{15} > 0\tag{6}$$

Things only physically crash once and that's the first time, so we choose the minus equation.

$$T_- = 100 - 20\sqrt{15} \text{ s} \simeq 22.5 \text{ s}\tag{7}$$

The cows witness a collision.

(b) If so, where will it take place?

To find where the crash takes place we just need to plug this time into one of our equations.

$$\begin{aligned}x_{\text{collision}} &= x_f(T_-) \\&= (15 \text{ m/s}) \times (22.5 \text{ s}) + 200 \text{ m} \\&= 538 \text{ m}\end{aligned}\tag{8}$$

(c) On a single graph, sketch the positions of the front of the passenger train and the back of the freight train. The graph is given below:

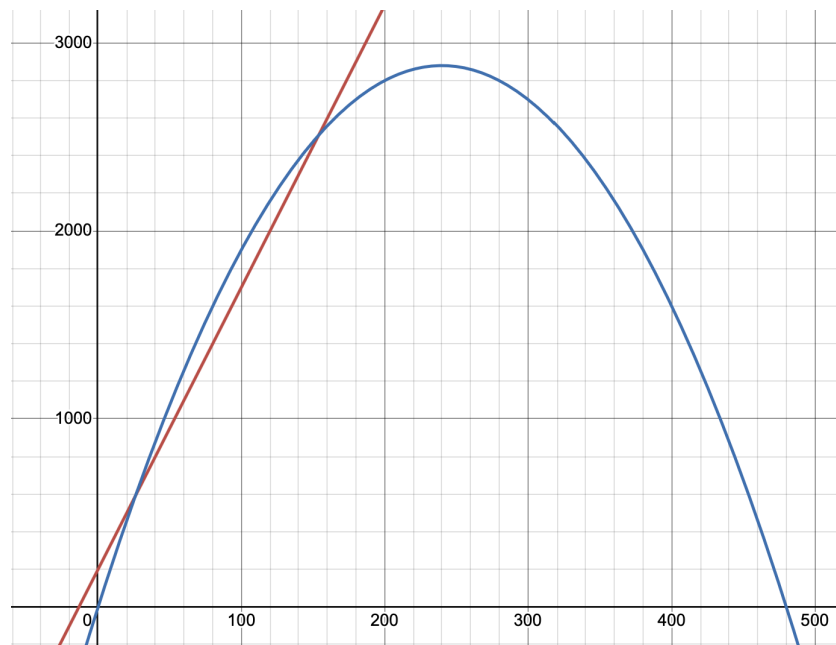


Figure 1: Time evolution of the two trains. The passenger train(blue) and the freight train(red) will collide at the first intersection.

Problem 2 (problem 2.68 on page 63 of the textbook)

CALC An object's velocity is measured to be $v_x(t) = \alpha - \beta t^2$, where $\alpha = 4.00$ m/s and $\beta = 2.00$ m/s³. At $t = 0$, the object is at $x = 0$.

(a) Calculate the object's position and acceleration as functions of time.

For this, we just need to take the derivative to get the acceleration and the integral to get the position.

$$v_x(t) = \alpha - \beta t^2 = (4.00 \text{ m/s}) - (2.00 \text{ m/s}^3) t^2 \quad (9)$$

$$\begin{aligned} a_x(t) &= \frac{dv_x(t)}{dt} \\ &= \frac{d}{dt}(\alpha - \beta t^2) \\ &= -2\beta t \end{aligned} \quad (10)$$

$$= (-4.00 \text{ m/s}^3) t \quad (11)$$

$$x(t) = \int v_x(t) dt \quad (12)$$

$$\begin{aligned} &= \int (\alpha - \beta t^2) dt \\ &= x(t=0) + \alpha t - \beta \frac{t^3}{3} \\ &= 0 \text{ m} + (4.00 \text{ m/s}) t - \left(\frac{2}{3} \text{ m/s}^3\right) t^3 \end{aligned} \quad (13)$$

(b) What is the object's maximum positive displacement from the origin?

So far as the velocity has a positive x direction, the positive displacement from the origin increases. Also, we know from problem (a) that the acceleration is always negative. Therefore, in order to find the maximum displacement, we need to find the time T when $v_x(t) = 0$ m/s and plug in the time to our position function.

$$v_x(t) = \alpha - \beta T^2 = 0 \text{ m/s} \quad (14)$$

$$\Rightarrow T = \sqrt{\frac{\alpha}{\beta}} = \sqrt{2} \text{ s} \quad (15)$$

Plugging in to Eq. (13), we get

$$\begin{aligned} x_{\text{Max}} &= x(T) \\ &= 0 \text{ m} + (4.00 \text{ m/s}) T - \left(\frac{2}{3} \text{ m/s}^3\right) T^3 \end{aligned} \quad (16)$$

$$= \frac{8\sqrt{2}}{3} \text{ m} \quad (17)$$

Problem 3 (A Multistage Rocket: problem 2.76 on page 63 of the textbook)

In the first stage of a two-stage rocket, the rocket is fired from the launch pad starting from rest but with a constant acceleration of 3.50 m/s^2 upward. At 25.0 s after launch, the second stage fires for 10.0 s , which boosts the rocket's velocity to 132.5 m/s upward at 35.0 s after launch. This firing uses up all of the fuel, however, so after the second stage has finished firing, the only force acting on the rocket is gravity. Ignore air resistance.

(a) Find the maximum height that the stage-two rocket reaches above the launch pad.

This problem is best solved by splitting it up into stages: Stage 1, Stage 2, and Stage 3. The v-t graph and the y-t graph is shown in Fig. 2 First, let's find the position and velocity of the rocket after stage one (25 seconds):

$$x_1 = \frac{3.5}{2}t_1^2 = \frac{3.5}{2}(25)^2 = 1093.8 \text{ m} \quad (18)$$

$$v_1 = 3.5t_1 = 3.5(25) = 87.5 \text{ m/s} \quad (19)$$

Now we can find the "average" acceleration of the second stage with these variables. 35 seconds after the rocket launches it is going at 132.5 m/s . So 10 seconds after the rocket is going 87.5 m/s it is going 132.5 .

$$v_2 = at_2 + 87.5 = 132.5 \text{ m/s} \quad (20)$$

Solving we get,

$$v_2 = 10a + 87.5 = 132.5 \text{ m/s} \quad (21)$$

$$\Rightarrow a = 4.5 \text{ m/s}^2 \quad (22)$$

The final position above the pad will be given by the position of the 3rd stage until the rocket stops moving upwards. So now we know our second and third position equations, solving the second one we get,

$$\begin{aligned} x_2 &= 1093.8 + 87.5t + \frac{4.5}{2}(t)^2 \\ &= 1093.8 + 87.5(10) + \frac{4.5}{2}(10)^2 \\ &= 2193.8 \text{ m} \end{aligned} \quad (23)$$

For the third stage, the only acceleration is gravity now so the position and velocity equations are,

$$p_3 = 2193 + 132.5t - \frac{9.81}{2}t^2 \quad (24)$$

$$v_3 = 132.5 - 9.81t \quad (25)$$

Solving for the max height we get,

$$t = 132.5/9.81 = 13.5 \text{ s} \quad (26)$$

Plugging in we get,

$$p_{\max} = 3088.6 \quad (27)$$

(b) How much time after the end of the stage-two firing will it take for the rocket to fall back to the launch pad? Solving for when the rocket meets the pad again we get,

$$3088.6 - \frac{9.8}{2}t^2 = 0 \quad (28)$$

$$t_{\text{fall}} = 25.1 \text{ s} \quad (29)$$

This is the time after the second stage so we need to account for the first part of the arc.

$$t_f = 25.1 + 13.5 = 38.6 \text{ s} \quad (30)$$

(c) How fast will the stage-two rocket be moving just as it reaches the launch pad?

Plugging in the time from the highest point to zero into the free fall equation we get,

$$-9.8t = v_f = -9.8(25.1) = 246 \text{ m/s} \quad (31)$$

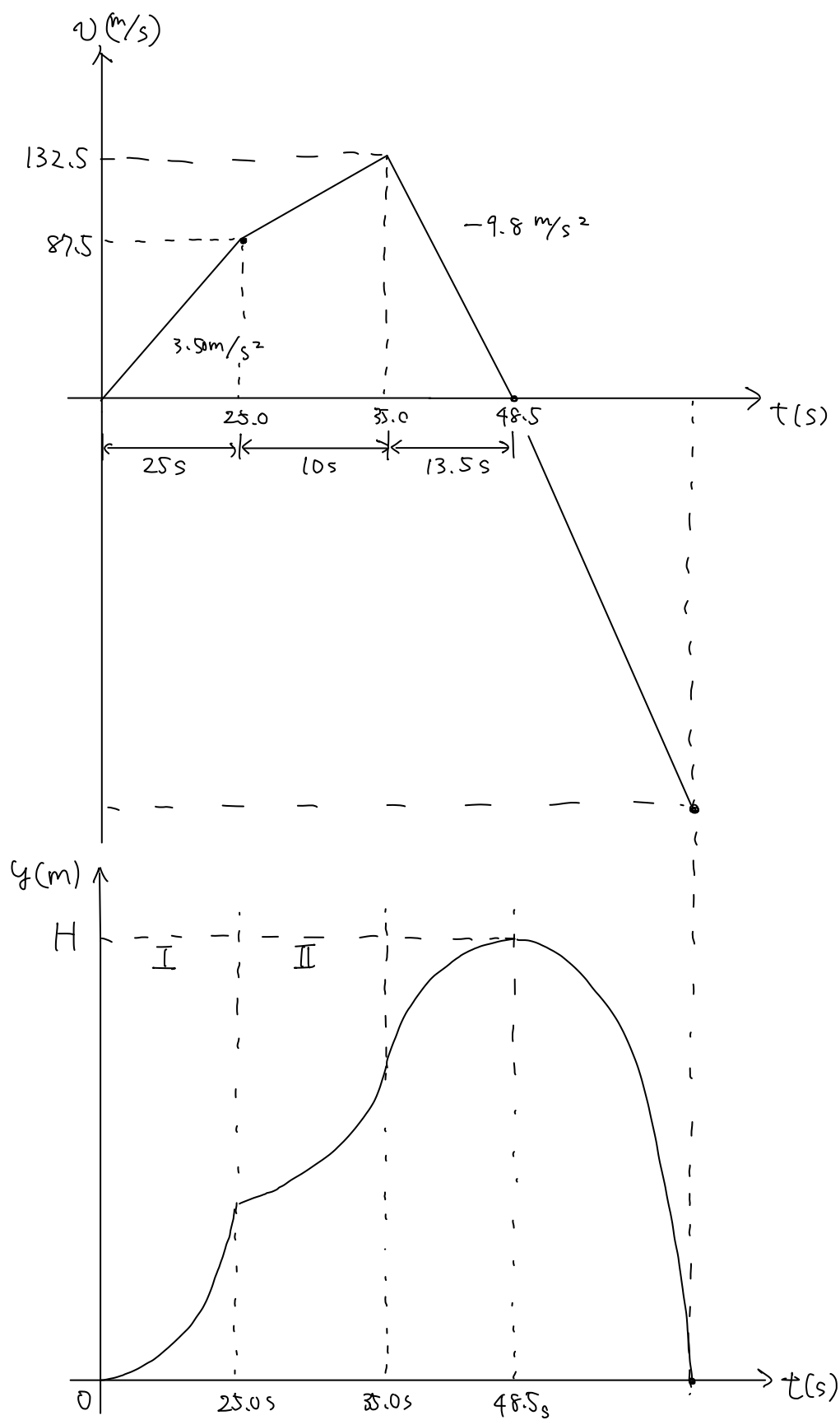


Figure 2: The v - t graph and the y - t graph of the two stage jet motion